6.7 Perform Similarity Transformations

Before You performed congruence transformations.

Now You will perform dilations.

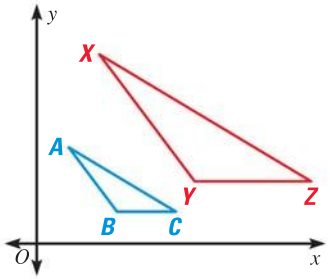
Why? So you can solve problems in art, as in Ex. 26.

Key Vocabulary
- **dilation**
- **center of dilation**
- **scale factor of a dilation**
- **reduction**
- **enlargement**
- **transformation,** *p. 272*

A **dilation** is a transformation that stretches or shrinks a figure to create a similar figure. A dilation is a type of *similarity transformation*.

In a dilation, a figure is enlarged or reduced with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** is the ratio of a side length of the image to the corresponding side length of the original figure. In the figure shown, $\triangle XYZ$ is the image of $\triangle ABC$. The center of dilation is (0, 0) and the scale factor is $\frac{XY}{AB}$.



KEY CONCEPT *For Your Notebook*

Coordinate Notation for a Dilation

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a **reduction**. If $k > 1$, the dilation is an **enlargement**.

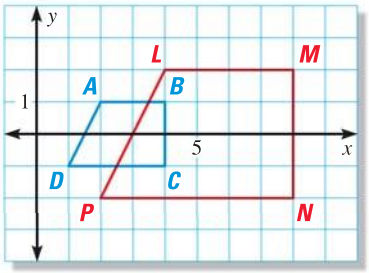
EXAMPLE 1 Draw a dilation with a scale factor greater than 1

READ DIAGRAMS
All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 1), *B*(4, 1), *C*(4, −1), and *D*(1, −1). Use a scale factor of 2.

Solution

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.



$$(x, y) \rightarrow (2x, 2y)$$

$A(2, 1) \rightarrow L(4, 2)$

$B(4, 1) \rightarrow M(8, 2)$

$C(4, -1) \rightarrow N(8, -2)$

$D(1, -1) \rightarrow P(2, -2)$

EXAMPLE 2 Verify that a figure is similar to its dilation

A triangle has the vertices $A(4, -4)$, $B(8, 2)$, and $C(8, -4)$. The image of $\triangle ABC$ after a dilation with a scale factor of $\frac{1}{2}$ is $\triangle DEF$.

a. Sketch $\triangle ABC$ and $\triangle DEF$.

b. Verify that $\triangle ABC$ and $\triangle DEF$ are similar.

Solution

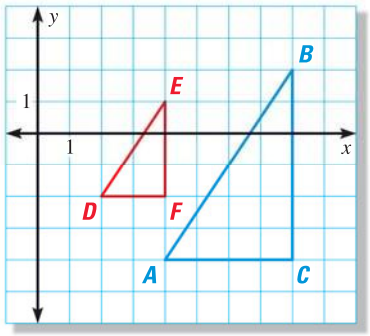
a. The scale factor is less than one, so the dilation is a reduction.

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$$A(4, -4) \rightarrow D(2, -2)$$

$$B(8, 2) \rightarrow E(4, 1)$$

$$C(8, -4) \rightarrow F(4, -2)$$



b. Because $\angle C$ and $\angle F$ are both right angles, $\angle C \cong \angle F$. Show that the lengths of the sides that include $\angle C$ and $\angle F$ are proportional. Find the horizontal and vertical lengths from the coordinate plane.

$$\frac{AC}{DF} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{2} = \frac{6}{3} \checkmark$$

So, the lengths of the sides that include $\angle C$ and $\angle F$ are proportional.

▶ Therefore, $\triangle ABC \sim \triangle DEF$ by the SAS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the coordinates of L, M, and N so that $\triangle LMN$ is a dilation of $\triangle PQR$ with a scale factor of k. Sketch $\triangle PQR$ and $\triangle LMN$.

1. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$

2. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$

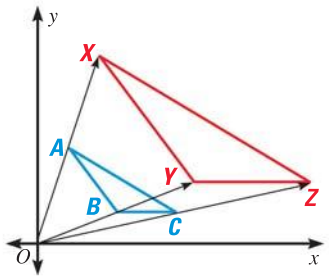
EXAMPLE 3 Find a scale factor

PHOTO STICKERS You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of the reduction?

Solution

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$. In simplest form, the scale factor is $\frac{11}{40}$.

READING DIAGRAMS Generally, for a center of dilation at the origin, a point of the figure and its image lie on the same ray from the origin. However, if a point of the figure *is* the origin, its image is also the origin.



EXAMPLE 4 Standardized Test Practice

You want to create a quadrilateral *EFGH* that is similar to quadrilateral *PQRS*. What are the coordinates of *H*?

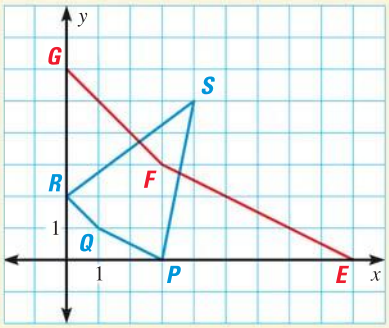
ELIMINATE CHOICES You can eliminate choice A, because you can tell by looking at the graph that *H* is in Quadrant I. The point (12, −15) is in Quadrant II.

Ⓐ (12, −15)

Ⓑ (7, 8)

Ⓒ (12, 15)

Ⓓ (15, 18)



Solution

Determine if *EFGH* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *E*, *F*, and *G* from *P*, *Q*, and *R*.

$(x, y) \rightarrow (kx, ky)$

$P(3, 0) \rightarrow E(9, 0)$ $\quad k = 3$

$Q(1, 1) \rightarrow F(3, 3)$ $\quad k = 3$

$R(0, 2) \rightarrow G(0, 6)$ $\quad k = 3$

Because k is the same in each case, the image is a dilation with a scale factor of 3. So, you can use the scale factor to find the image *H* of point *S*.

$S(4, 5) \rightarrow H(3 \cdot 4, 3 \cdot 5) = H(12, 15)$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

CHECK Draw rays from the origin through each point and its image.

GUIDED PRACTICE for Examples 3 and 4

3. **WHAT IF?** In Example 3, what is the scale factor of the reduction if your photo is 5.5 inches by 5.5 inches?

4. Suppose a figure containing the origin is dilated. *Explain* why the corresponding point in the image of the figure is also the origin.

6.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 11, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 21, 22, 28, 30, and 31

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In a dilation, the image is __?__ to the original figure.

2. ★ **WRITING** *Explain* how to find the scale factor of a dilation. How do you know whether a dilation is an enlargement or a reduction?

EXAMPLES 1 and 2 on pp. 409–410 for Exs. 3–8

DRAWING DILATIONS **Draw a dilation of the polygon with the given vertices using the given scale factor *k*.**

3. $A(-2, 1), B(-4, 1), C(-2, 4); k = 2$

4. $A(-5, 5), B(-5, -10), C(10, 0); k = \frac{3}{5}$

5. $A(1, 1), B(6, 1), C(6, 3); k = 1.5$

6. $A(2, 8), B(8, 8), C(16, 4); k = 0.25$

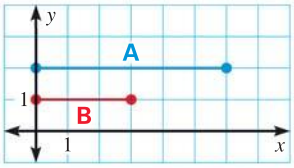
7. $A(-8, 0), B(0, 8), C(4, 0), D(0, -4); k = \frac{3}{8}$

8. $A(0, 0), B(0, 3), C(2, 4), D(2, -1); k = \frac{13}{2}$

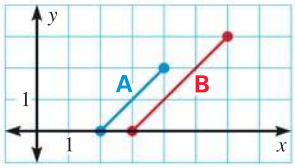
EXAMPLE 3 on p. 410 for Exs. 9–12

IDENTIFYING DILATIONS **Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.**

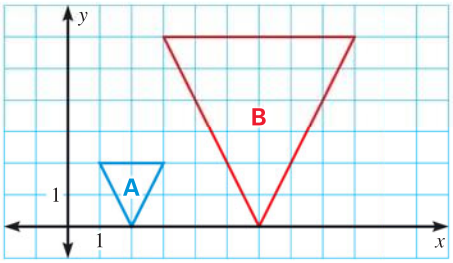
9.



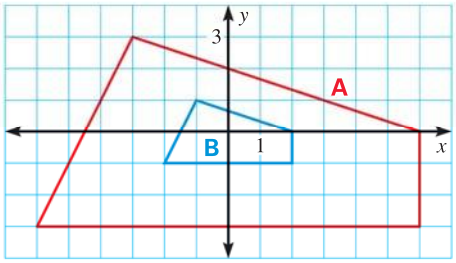
10.



11.

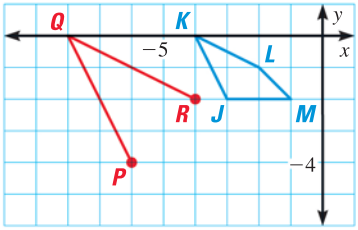


12.



EXAMPLE 4 on p. 411 for Ex. 13

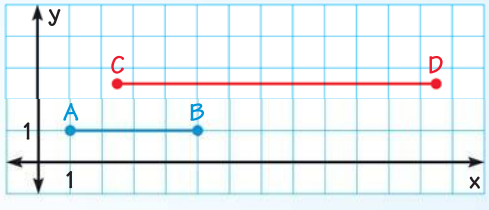
13. ★ **MULTIPLE CHOICE** You want to create a quadrilateral *PQRS* that is similar to quadrilateral *JKLM*. What are the coordinates of *S*?



Ⓐ (2, 4) Ⓑ (4, −2)

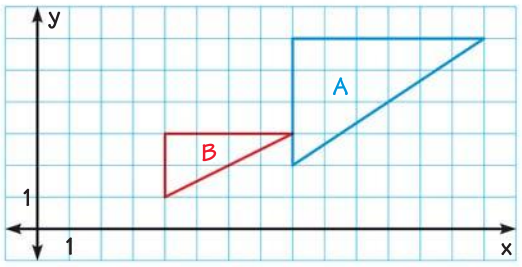
Ⓒ (−2, −4) Ⓓ (−4, −2)

14. **ERROR ANALYSIS** A student found the scale factor of the dilation from $\overline{AB}$ to $\overline{CD}$ to be $\frac{2}{5}$. *Describe* and correct the student's error.



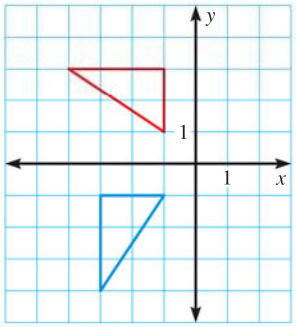
$\frac{AB}{CD} = \frac{2}{5}$

15. ERROR ANALYSIS A student says that the figure shown represents a dilation. What is wrong with this statement?

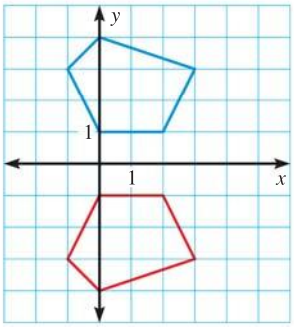


IDENTIFYING TRANSFORMATIONS **Determine whether the transformation shown is a *translation*, *reflection*, *rotation*, or *dilation*.**

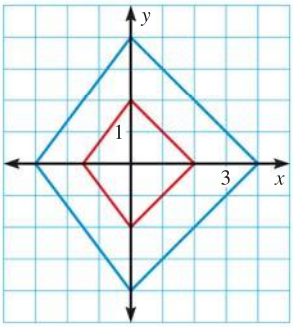
16.



17.

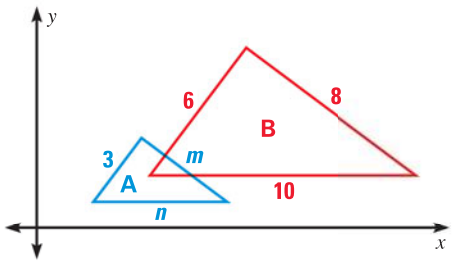


18.

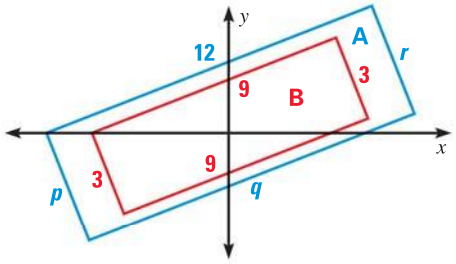


FINDING SCALE FACTORS **Find the scale factor of the dilation of Figure A to Figure B. Then give the unknown lengths of Figure A.**

19.



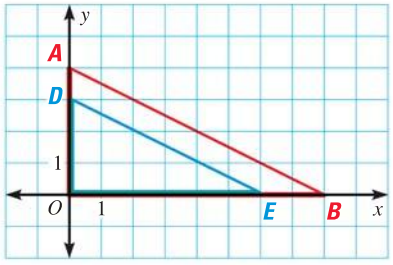
20.



21. ★ MULTIPLE CHOICE In the diagram shown, $\triangle ABO$ is a dilation of $\triangle DEO$. The length of a median of $\triangle ABO$ is what percent of the length of the corresponding median of $\triangle DEO$?

Ⓐ 50% Ⓑ 75%

Ⓒ $133\frac{1}{3}\%$ Ⓓ 200%



22. ★ SHORT RESPONSE Suppose you dilate a figure using a scale factor of 2. Then, you dilate the image using a scale factor of $\frac{1}{2}$. *Describe* the size and shape of this new image.

CHALLENGE ***Describe*** **the two transformations, the first followed by the second, that combined will transform $\triangle ABC$ into $\triangle DEF$.**

23. $A(-3, 3)$, $B(-3, 1)$, $C(0, 1)$
$D(6, 6)$, $E(6, 2)$, $F(0, 2)$

24. $A(6, 0)$, $B(9, 6)$, $C(12, 6)$
$D(0, 3)$, $E(1, 5)$, $F(2, 5)$

PROBLEM SOLVING

EXAMPLE 3 on p. 410 for Exs. 25–27

25. BILLBOARD ADVERTISEMENT A billboard advertising agency requires each advertisement to be drawn so that it fits in a 12-inch by 6-inch rectangle. The agency uses a scale factor of 24 to enlarge the advertisement to create the billboard. What are the dimensions of a billboard, in feet?

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26. POTTERY Your pottery is used on a poster for a student art show. You want to make postcards using the same image. On the poster, the image is 8 inches in width and 6 inches in height. If the image on the postcard can be 5 inches wide, what scale should you use for the image on the postcard?

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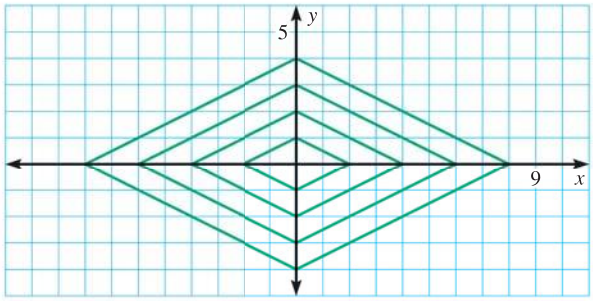
27. SHADOWS You and your friend are walking at night. You point a flashlight at your friend, and your friend's shadow is cast on the building behind him. The shadow is an enlargement, and is 15 feet tall. Your friend is 6 feet tall. What is the scale factor of the enlargement?

28. ★ OPEN-ENDED MATH *Describe* how you can use dilations to create the figure shown below.



AnimatedGeometry at classzone.com

29. MULTI-STEP PROBLEM $\triangle ABC$ has vertices $A(3, -3)$, $B(3, 6)$, and $C(15, 6)$.

a. Draw a dilation of $\triangle ABC$ using a scale factor of $\frac{2}{3}$.

b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?

c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?

30. ★ EXTENDED RESPONSE Look at the coordinate notation for a dilation on page 409. Suppose the definition of dilation allowed $k < 0$.

a. *Describe* the dilation if $-1 < k < 0$.

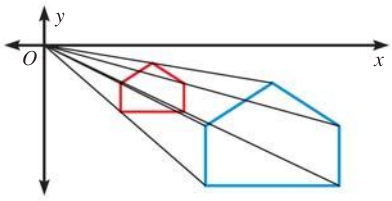
b. *Describe* the dilation if $k < -1$.

c. Use a rotation to describe a dilation with $k = -1$.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

31. ★ **SHORT RESPONSE** *Explain* how you can use dilations to make a perspective drawing with the center of dilation as a vanishing point. Draw a diagram.



32. **MIDPOINTS** Let $\overline{XY}$ be a dilation of $\overline{PQ}$ with scale factor k. Show that the image of the midpoint of $\overline{PQ}$ is the midpoint of $\overline{XY}$.

33. **REASONING** In Exercise 32, show that $\overline{XY} \parallel \overline{PQ}$.

34. **CHALLENGE** A rectangle has vertices $A(0, 0)$, $B(0, 6)$, $C(9, 6)$, and $D(9, 0)$. *Explain* how to dilate the rectangle to produce an image whose area is twice the area of the original rectangle. Make a conjecture about how to dilate any polygon to produce an image whose area is n times the area of the original polygon.

MIXED REVIEW

Simplify the expression. *(p. 873)*

35. $(3x + 2)^2 + (x - 5)^2$ **36.** $4\left(\frac{1}{2}ab\right) + (b - a)^2$ **37.** $(a + b)^2 - (a - b)^2$

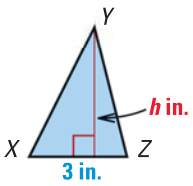
Find the distance between each pair of points. *(p. 15)*

38. (0, 5) and (4, 3) **39.** (−3, 0) and (2, 4) **40.** (−2, −4) and (3, −2)

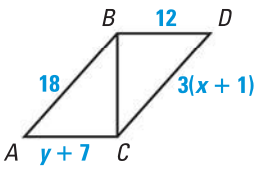
PREVIEW
Prepare for Lesson 7.1 in Exs. 41–43.

Find the value(s) of the variable(s).

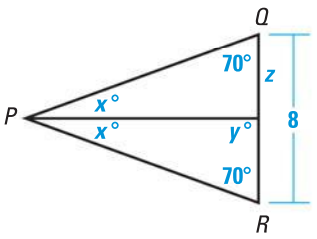
41. Area = 6 in.^{2} *(p. 49)*



42. $\triangle ABC \cong \triangle DCB$ *(p. 256)*



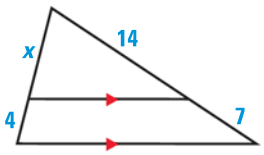
43. $\triangle PQR$ is isosceles. *(p. 303)*



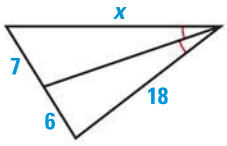
QUIZ *for Lessons 6.6–6.7*

Find the value of x. *(p. 397)*

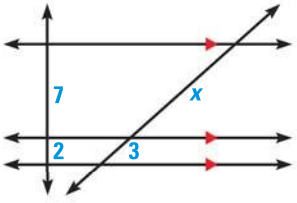
1.



2.



3.



Draw a dilation of $\triangle ABC$ with the given vertices and scale factor k. *(p. 409)*

4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = 0.4$ **5.** $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2.5$